

Gap Generation in Topological Insulator Surface States by non-Ferromagnetic Magnets

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It is shown that, contrary to the naive expectation, single particle spectral gaps can be opened on the surface states of three dimensional topological insulators by using commensurate out- and in-plane antiferromagnetic or ferrimagnetic insulating thin films.

PACS numbers: 75.70-i, 75.30.GW, 73.20.-r, 85.75.-d

Introduction. One of the most remarkable properties of a three dimensional topological insulator is the presence of a topologically quantized magnetoelectric term (TMET) in its electromagnetic response. This term has far reaching consequences since it constitutes a condensed-matter realization of axion electrodynamics[1, 2]. Experimental signatures of the TMET include the quantized Kerr angle and Faraday rotation[3–5], Casimir repulsion[6], inverse spin galvanic effect[7], monopole images[8], surface half integer Hall effect[9], topological viscoelastic response[10] just to name a few.

The key point for the observability of this topologically quantized response is the breakdown of the time reversal symmetry in the surface of the otherwise time reversal invariant TI[9]. In terms of the electric and magnetic fields, the TMET in the electromagnetic action takes the form

$$S_\theta = \frac{\alpha}{4\pi^2} \int d^3\mathbf{r} d\theta \mathbf{E} \cdot \mathbf{B}, \quad (1)$$

where α is the fine structure constant and θ is the so called axion parameter which takes the value of 0 or $(2n+1)\pi$ with $n \in \mathbb{N}$ in trivial and topological insulators, respectively[9]. Alternatively to the above description in terms of the electromagnetic fields, one can understand the TMET as a Chern Simons (CS) term induced in the electromagnetic response of the insulator by the gapped surface states of a TI that are described by usual massive Dirac Hamiltonian:

$$H_D = v(\boldsymbol{\sigma} \times \mathbf{k}) \cdot \hat{\mathbf{z}} + m\sigma_z, \quad (2)$$

where v is the Fermi velocity, and m is the induced mass of the Dirac states. In this case, the value $\theta = \pi$ corresponds to the value $\sigma = \frac{1}{2}\text{sign}(m)$ for the Hall coefficient in the corresponding CS term. In short, breaking time reversal symmetry opens a gap in the TI surface states, thus making the TMET observable.

Within the effective low energy approximation described by (2) there are several proposals in the literature for opening a gap in the helical metal by means of weak magnetic fields through a Zeeman term $H_Z =$

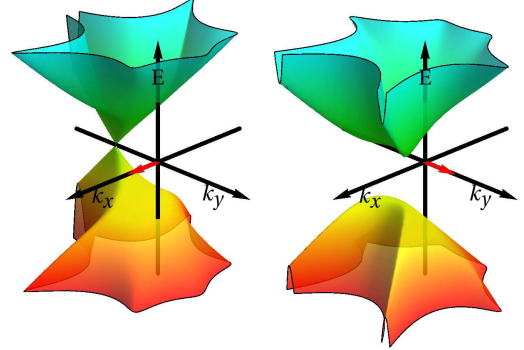


FIG. 1: (Color online). Low energy spectrum of the surface states of a TI in homogeneous in-plane magnetization. Red arrows show the direction of the magnetization field.

$g\mu_B\boldsymbol{\sigma} \cdot \mathbf{B}$ [11], or through exchange coupling to ferromagnetic thin films $H_{exc} = J\mathbf{M} \cdot \boldsymbol{\sigma}$ [9], and magnetic impurities $H_{imp} = J\sum_j \mathbf{S}_j \cdot \boldsymbol{\sigma}\delta(\mathbf{r} - \mathbf{R}_j)$ [12–14]. The exchange coupling between magnetic thin films and TI is the most appealing from the theoretical point of view because it not only gives a simple mechanism to develop the theory of the TMET but it allow us to look for unexpected effects that can alter the thin film magnetization dynamics[7, 15]. However, this proposal is experimentally challenging, and also it poses some questions. First of all, it is not so easy to find insulating ferromagnetic materials. Some candidate materials like GdN and EuO have been theoretically suggested[3, 7] but to the best of our knowledge so far there is no experimental evidence supporting this claim. Also, even if ferromagnetic insulating thin films were available, it is not guaranteed that the thin film magnetization would point in the out-of-plane direction[16]. There is the problem of possible mismatch between the TI surface lattice structure and the thin film lattice structure and even the issue of the two lattices not being commensurate. These problems are in the heart of the experimental difficulties for implementing this mechanism.

Directly using eq.(2) implicitly forces us to consider a continuum medium approach for the magnetization [17]. The question is then how to construct this effective

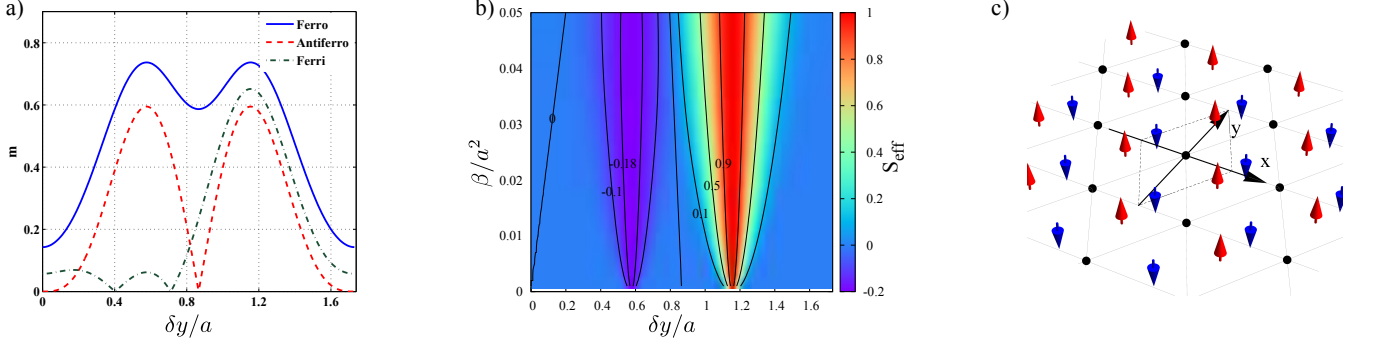


FIG. 2: a): Exchange induced gap m (in units of B_{11}) on the TI surface states vs. the lattice mismatch $\delta \mathbf{y}$. Blue (solid), red (dashed), and green (dashed-dotted) lines corresponds to ferromagnetic, antiferromagnetic, and ferrimagnetic lattices, respectively. For the ferrimagnetic configuration $\mathbf{S}_1 = -5\mathbf{S}_2$ and $\beta = 0.02a^2$. b): The effective Zeeman field on the surface as the function of the relative displacement $\delta \mathbf{y}$ between the magnetic and the TI lattices and the localization parameter β . c): Real space configurations for the TI surface. Black dots represent lattice Se positions and the arrows correspond to a hexagonal antiferromagnetic lattice.

description of the exchange coupling between the surface electronic spin and the magnetization starting from a microscopic model. For ideal insulating ferromagnets the most naive way would be to couple the electron spin with the averaged magnetization in the magnetic unit cell. However when more realistic magnetic insulators are considered we immediately run into difficulties. For instance this approach automatically rules out the possibility of considering antiferromagnetic insulators as magnetic material candidates. Also it is not at all clear what the correct form for a continuum description of the magnetization of a ferrimagnetic insulator is. Motivated by these experimental and theoretical issues, we address in this work the problem of coupling a magnetically active thin film with the surface electronic states of a TI employing an tight binding approach.

The model. In order to study qualitatively the ways in which the gap can be microscopically induced in the surface spectrum we will employ a tight binding model valid for the topological insulators of the form Bi_2X_3 which include the prototypical examples of TI's Bi_2Se_3 and Bi_2Te_3 . We will then follow references [18, 19] and will consider a Bi_2Se_3 sample made of N quintuple layers (QL) grown in the (111) direction and terminated in Se planes. The surface will thus have a triangular lattice structure. We are interested in the bandstructure around the Fermi level so the tight-binding basis set will be made of linear combinations of the atomic orbitals $(|p_{\text{Bi}}, \uparrow\rangle, |p_{\text{Se}}, \uparrow\rangle, |p_{\text{Bi}}, \downarrow\rangle, |p_{\text{Se}}, \downarrow\rangle)$. The superscript reflect the parity of the state and the second index is the spin polarization. The tight binding Hamiltonian in real space can be written in this basis in the following way[18, 19]:

$$H = \sum_{\mathbf{n}} C_{\mathbf{n}}^\dagger \hat{\epsilon} C_{\mathbf{n}} + \sum_{\mathbf{n}, \mathbf{a}_i/\mathbf{b}_i} C_{\mathbf{n}}^\dagger \hat{t}_{\mathbf{a}_i/\mathbf{b}_i} C_{\mathbf{n}+\mathbf{a}_i/\mathbf{b}_i} + H.c. \quad (3)$$

Here the lattice vectors \mathbf{a}_i and \mathbf{b}_i connect unit cell positions within the same QL and of different QL, respectively and \mathbf{n} labels the lattice positions as defined in [18, 19]. We use $a = |\mathbf{a}_i|$ as the lateral spatial length scale. The on-site energy $\hat{\epsilon}$ and hopping terms $\hat{t}_{\mathbf{a}_i/\mathbf{b}_i}$ are 4×4 matrices which can be written as linear combination of Γ_i matrices which are matrix products of spin $\boldsymbol{\sigma}$ and parity $\boldsymbol{\tau}$ Pauli matrices:

$$\begin{aligned} \hat{\epsilon} &= \epsilon_0 \Gamma_0 + m \Gamma_5, \\ \hat{t}_{\mathbf{a}_1} &= A_0 \Gamma_0 - i(A_{12} \Gamma_3 - A_{14} \Gamma_2) + A_{11} \Gamma_5, \\ \hat{t}_{\mathbf{b}_1} &= B_0 \Gamma_0 + i(B_{12} \Gamma_4 - B_{14} \Gamma_1) + B_{11} \Gamma_5, \\ \Gamma_{1,2} &= \tau_1 \otimes \sigma_{1,2}, \Gamma_3 = \tau_1 \otimes \sigma_3, \\ \Gamma_{4,5} &= \tau_{2,3} \otimes \sigma_0, \Gamma_0 = \tau_0 \otimes \sigma_0. \end{aligned} \quad (4)$$

The remaining hopping matrices $\hat{t}_{\mathbf{a}_{2,3}/\mathbf{b}_{2,3}}$ can be obtained from (4) by applying the rotation operation $R_3 = \exp(i\frac{\pi}{3}\sigma_3 \otimes \tau_0)$. The Hamiltonian (3) is thus made of intra-QL hopping terms and on-site energies and hopping terms coupling different QL. In all calculations presented here we use $B_{11} = 1$, $A_{14} = 1.4$, $A_{12} = B_{12} = 3$, $A_{11} = 2$, $m = -10$, $B_{14} = A_0 = B_0 = 0$, for modelling a bulk TI[19]. Next we add a exchange term coupling to the Se atomic orbitals in (3) of the first QL of the form

$$H_{\text{exc}} = J \sum_{\mathbf{n}} \mathbf{S}(\mathbf{R}_{\mathbf{n}}) C_{\mathbf{n}}^\dagger \boldsymbol{\Sigma} C_{\mathbf{n}}. \quad (5)$$

where The matrices $\boldsymbol{\Sigma}$ are of the form $\boldsymbol{\Sigma} = \frac{1}{2}(\tau_0 - \tau_3) \otimes \boldsymbol{\sigma}$ and now $\mathbf{R}_{\mathbf{n}}$ represent the lattice positions of the Se atoms in the outer part of the first QL. The important observation here is that the magnetic and surface lattices *do not need to be the same for generic magnetic layers* so the magnetic moment of the magnetic layer $\mathbf{S}(\mathbf{R}_{\mathbf{n}})$ at $\mathbf{R}_{\mathbf{n}}$ will not be the magnetic moment of

each magnetic position. Usually the magnetic moments represent the magnetic moment associated to a bounded atomic orbital with a short spatial extension so not all the magnetic moments will couple in the same manner to the electronic spins on the surface and the coupling will be stronger for nearer atoms. The two previous observations lead us to define $\mathbf{S}(\mathbf{R}_n)$ as

$$\mathbf{S}(\mathbf{R}_n) \equiv \sum_i \mathbf{S}(\hat{\mathbf{R}}_i) \Phi(\mathbf{R}_n - \hat{\mathbf{R}}_i), \quad (6)$$

where now the sum is performed over the magnetic lattice positions. The function Φ encodes the information about the short range character of the localized magnetic orbitals. In our calculations we have chosen a Gaussian profile, $\Phi(\mathbf{r}) = e^{-\mathbf{r}^2/\beta}$ parametrized by the parameter β which has the meaning of the (squared) mean size of the spatial profile of the magnetic orbital. We have checked that any other choice for Φ does not modify the qualitative results presented in this work. It is important to note that for a given Se position, nearby moments will contribute to $\mathbf{S}(\mathbf{R}_n)$ but not equally if there is a mismatch between the two sublattices. This key observation is interesting because it opens the possibility of considering not just ferromagnetic, but also other types of magnetic ordering as a candidate for inducing gaps in the TI surface states by the exchange coupling mechanism.

Results. In order to show the ideas explained above at work, let us consider first the case where the positions of the magnetic lattice lie in the middle of the triangles formed by the surface lattice as it is shown in Fig.(2c) and calculate the spectrum with eqs (3-6). We will control the mismatch between the lattices by displacing a magnetic bipartite lattice a distance $\delta\mathbf{y}$ with respect the center of the triangle in the OY direction, and we will consider the three cases of ferromagnetic ($\mathbf{S}_1 = \mathbf{S}_2$), antiferromagnetic ($\mathbf{S}_1 = -\mathbf{S}_2$) and ferrimagnetic ($\mathbf{S}_1 = -5\mathbf{S}_2$) out of plane configurations. In Fig.(2a) we show the value of the gap defined as $m = |\min[E_c(\mathbf{k})] - \max[E_v(\mathbf{k})]|/2$. For the ferromagnetic case, the system always develops a non zero gap, as expected, irrespective of the relative position of the two lattice sites. The modulation in the value of the gap is understood in terms of the different contribution of the magnetic moments to $\mathbf{S}_{eff}(\mathbf{R}_j)$. Much more interesting are the cases of ferrimagnetic and antiferromagnetic lattice structures. The first important observation is that in both cases a gap is opened when varying the relative position of the lattices, showing that in principle one can open gaps in the TI surface states by the interaction with ferri- and antiferromagnetic layers. In principle, nothing guarantees that the magnetic lattice sites must lie on the exact center of the triangles formed by the surface positions, but the gap might be still open. Moreover, if during the fabrication process it were possible to control the lattice mismatch, the gap could be tuned. Another important observation is that although m is a positive definite quantity by construc-

tion, the value of the effective Zeeman coupling is not. Indeed it will change its sign, as it is shown in fig.(2b), where the effective Zeeman term is plotted as a function of the lattice displacement $\delta\mathbf{y}$ and the value of β . This result means that for the case of ferrimagnet different relative displacements might lead to different values of the coupling, which is the signature of a topological phase transition, controlled by $\delta\mathbf{y}$. This is also true for generic antiferromagnetic configurations. As can be readily seen in fig.(2b) there is always a change of sign of \mathbf{S}_{eff} irrespective of how tight are the magnetic atomic orbitals to their lattice sites.

So far, we have considered magnetic configurations in thin layers with the magnetization being out-of-plane. It is well known that when thin film geometries are considered for ferromagnets it is more energetically favorable for the system to have the magnetization in plane[16, 20]. From the form of (2) an in-plane homogeneous magnetization would not induce any gap since an in plane magnetic moment would just shift the position of the Dirac point. Actually this is not the case and a gap can be induced when lattice effects are considered in addition to (2) as it was shown by Fu[21]. We can add to (2) the two next to leading terms in the expansion in momenta[21]:

$$H_w = \frac{\mathbf{k}^2}{2m_0} \sigma_0 + \alpha \mathbf{k}^2 (\boldsymbol{\sigma} \times \mathbf{k}) \cdot \hat{\mathbf{z}} + \lambda (\mathbf{k}_x^3 - 3k_x k_y^2) \sigma_z, \quad (7)$$

where m_0 , α , and λ come from the comparison between the band structure calculated from the tight binding and ARPES measurements[22]. When the Hamiltonian $H_D + H_w$ is considered together with $H_{exc} = J_{||} \boldsymbol{\sigma}_{||} \mathbf{m}_{||}$, it is apparent that a gap of value $\tilde{m} = \lambda \frac{J_{||}^3}{v^3} (m_y^3 - 3m_y m_x^2)$ appears at $\mathbf{k}_g = \frac{J_{||}}{v} \mathbf{m}_{||} \times \hat{\mathbf{z}}$ as it is shown in fig.(1). Apart from the mass generation due to the hexagonal terms there is a self-doping effect defined through the parameter $\mu = |\min[E_c(\mathbf{k})] + \max[E_v(\mathbf{k})]|/2$ due to the first term in (7). The effective model $H_D + H_w + H_{exc}$ can be considered as a good description for the interaction between the TI surface states and smooth varying ferromagnetic in-plane magnetization. However, as we argued before there are many materials where the magnetization varies at the order of the lattice spacing and the above effective description cannot be directly applied. Even in such cases, a non vanishing gap can be found if one goes to the microscopic description of the system.

We will exemplify this situation considering the Kagome lattice with classical planar magnetic configurations. The magnetism on this frustrated lattice is a current subject of research[23]. In particular we have chosen the $q = 0$ and $q = \sqrt{3} \times \sqrt{3}$ ground state spin configurations as plotted in the insets of Fig.(3). In order to monitor the evolution of the spectral properties of the system we have chosen the lateral displacement $\delta\mathbf{x}$ between lattices in the OX direction as a control parameter. In this way the magnetic moment sitting on the

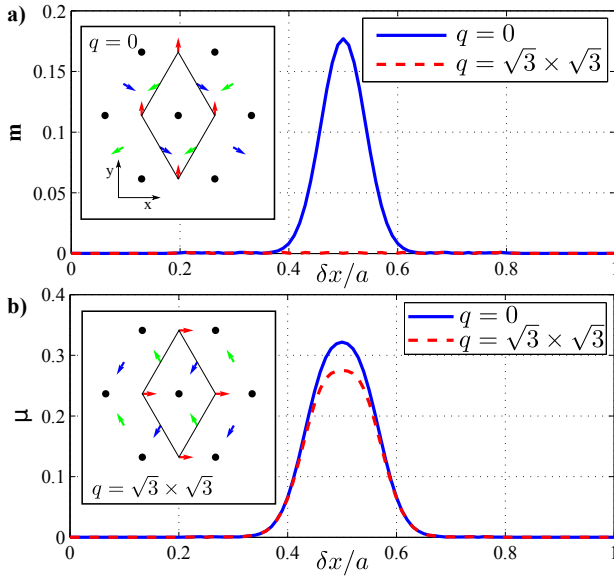


FIG. 3: Evolution of the gap a) and the self doping b) as a function of the mismatch $\delta\mathbf{x}$ between the two sublattices, as it is explained in the text. Insets correspond to magnetic configurations with $\delta\mathbf{x} = 0$.

horizontal axis will play a dominant role. The results for the gap and for the self doping are also displayed in Fig.(3). We can easily understand the results by keeping in mind that the exchange interaction between magnetic moments and electron spins is short ranged and the behavior of the gap with the magnetic moment components according to the Fu's model. In the inset of fig.(3a) the red sublattice magnetization points along the OY direction so the gap will open when this sublattice magnetization is closest to the atomic lattice. On the contrary, in the inset of Fig.(3b) the red sublattice magnetization points along the OX axis so according Fu's model, no gap will be generated. Also, it is expected that both configurations will give rise to a non vanishing self doping effect when the effective magnetization is non zero as it is shown in Fig.(3b). Another important observation is that different in-plane magnetic configurations in adjacent space regions (Néel domain walls[24]) might induce a mass with opposite sign which would generate chiral 1D fermionic states.

Experimental feasibility. One of the proposed ferromagnetic insulators is the EuO[25]. It possesses a gap of the order of 1.2eV and it crystallizes in the simple cubic structure, not commensurate to the triangular lattice structure of the surface, introducing further complexity in the problem. In contrast to ferromagnetic insulators, ferrimagnetic insulators offer more reliable experimental opportunities. The ferrimagnetic insulating state is present in Nature in many compounds and in many crystalline structures, and ferrimagnetic thin films

can be manufactured in many ways[26]. Among them we highlight the hexagonal ferrites of which $\text{PbFe}_{12}\text{O}_{19}$ is the archetypal material. They crystallize in the hexagonal magnetoplumbite structure having a rather complex atomic configuration. Many other ferrites grow in the spinel structure, like the magnetites (Fe_3O_4) and Cobalt ferrites (CoFe_2O_4) that might be grown in thin films with appreciable out-of-plane magnetization[27]. Although CoFe_2O_4 have a strong mismatch between the magnetic and the Se lattice structure and the results presented here are not directly applicable we suggest it as prospect candidate for experimentally analyzing the effect of ferrimagnetism on the surface states of a TI. Concerning in-plane magnetic configurations, we can mention Kagome systems with different planar spin ground states like $\text{SrCr}_9\text{Ga}_3\text{O}_{19}$, herbersmithite, jarosite and many others[28].

Conclusions. In the present paper we have addressed the question if the effective Hamiltonian (2) is valid when the helical surface states of a TI are coupled to magnetically active layers. By using a tight-binding model for both the TI and the magnetization, we have shown that contrary to the (perhaps too) naive expectation that the helical spin couples to the total magnetization present in the unit cell, it couples to a *weighted average* of the magnetic moments present in the unit cell. This result tell us that in principle, there is no physical reason for ruling out antiferromagnetic insulating thin films as candidates for inducing gaps in TI surface states. We have considered also the possibility of ferrimagnetic insulating thin films. In all the cases, we have shown that the magnetic exchange mechanism induces a gap in these surface states. As a result, we have found that the gap is sensitive to the mismatch between the magnetic and surface lattices. We have considered also the realistic situation where the film magnetization is in-plane and homogeneous. In this case, a gap might be opened due to hexagonal warping effects[21] even for materials whose thin film magnetization is textured at the scale of the lattice spacing, inducing a zero average magnetic moment per unit cell.

The authors gratefully acknowledge conversations with Carlos Pecharrmán, András Pályi and József Cserti. A. C. acknowledges the CSIC JAE-doc fellowship program for financial support. O. L. acknowledges the support of the EU grant NanoCTM.

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